THE EFFECT OF TRANSFORMATIONS ON THE ANALYSIS OF VARIANCE

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ABSTRACT

Nine transformations were studied for effect on the analysis of variance. They were ARC TANGENT, $\sqrt{x}$, $\sqrt{x} + \sqrt{x+1}$, LOG_{10}, EXP(x), INVERSE, LOG_e, SINE, and HYPERBOLIC TANGENT.

The factorial experiment used as a basis for this study was a three factor factorial with three levels per factor and three observations per cell. One effect was altered by the addition of a constant to provide one effect with a high percentage of significant records.

The transformations were applied to observations distributed according to the following distributions: Beta, Binomial, Erlang, Exponential, Gamma, Log-normal, Normal, Poisson, Uniform, and actual data from highway accident damage costs. For all distributions except the highway accident damage costs (8436), 10,000 experiments were studied.

When analysis of variance results were compared (before and after transformation), $\sqrt{x}$ and $\sqrt{x} + \sqrt{x+1}$ consistently changed the fewest results. All other transformations produced unstable results across the range of distributions, showing little effect to sometimes drastic change according to the distribution involved.

INTRODUCTION

It has been common practice for many years to transform raw data prior to performing an analysis of variance in order that the assumptions of the analysis of variance may be more accorately approximated; for example, see Davies (1954), or Fryer (1966).

In addition, it is well known that the analysis of variance tends to be robust, see Norton (1952), under some conditions. If robustness could be expected under all conditions, a study of the effects of transformations of raw data would be pointless.

In a computer aided review of pertinent literature, no single study was found concerning the impact of various transformations on the analysis of variance. Yet the practice of transforming of raw data is so common as to have been made available in various computer statistical packages; for example, Biomedical Statistical Package, (Dixon, 1973).

The purpose of this study was to examine the impact of various common statistical distributions at the .01 and .05 levels of significance of the analysis of variance. The transformations selected were ARC TANGENT, $\sqrt{x}$, $\sqrt{x} + \sqrt{x+1}$, LOG_{10}, EXP(x), INVERSE, LOG_e, SINE, and HYPERBOLIC TANGENT, selected because of availability in BMD (Dixon, 1973). The statistical distributions were Beta, Binomial, Erlang, Exponential, Gamma, Log-normal, Normal, Poisson, and Uniform, selected on the basis of being available in SIMSCRIPT. In addition to add a practical flavor to the problem, Alabama Highway Department Accident Damage Cost[1] data was included.

The factorial experiment used as a basis for this study was a three factor (A, B, C) factorial with three levels per factor and three observations per cell. This experiment was selected for the following reasons:

(1) an ABC design would provide two and three way interaction effects,

(2) three observations per cell would tend to provide "stability" in the cell,

(3) eighty-one observations would provide 54 degrees of freedom for test purposes (all factors were assumed fixed),

(4) increasing the number of factors would materially increase the number of computations without materially affecting the outcome, and

(5) increasing the number of observations per cell would materially increase the number of computations without materially affecting the outcome.

It was decided to force real significance[2] on one of the effects in the experiment so that one effect would have a relatively large percentage of significant records. By adding a constant to one level of a factor, the population mean of that level would be changed, forcing real significance. The third level of factor A was thus changed, except in the case of Highway (10). The individual constants added are specified in the discussion of each distribution.

Because it is not possible to perform all of the transformations on a value of zero, all remaining zero values (after the addition of the constant) were changed to 1.0. The only two distributions affected by this were the Binomial (02) and Poisson (08). The exact number of values so changed is specified in the individual distribution discussion.

The value ranges for the distributions were purposefully kept small because of the magnitude of values generated by transformation 06 (EXP(x)).

[1]David B. Brown, Personal Commun., 1975
[2]William D. Spears, Personal Commun., 1975

This is a restriction imposed by IBM 370 hardware. Any value over 18 was changed to 18. Not all distributions had values requiring this change but all are noted in the distribution discussions.

BETA DISTRIBUTION RESULTS

A random variable X is said to be distributed according to the Beta distribution of its density function is given by

$$f(x) = \frac{\Gamma(\alpha + \beta + 2)}{\Gamma(\alpha + 1)\Gamma(\beta + 1)} X^{\alpha}(1-x)^{\beta} \quad 0<x<1$$

where α and β are parameters with $\alpha > -1$ and $\beta > -1$ (Beyer, 1966, p. 18). The parameters used were $\alpha = 2$ and $\beta = 2$. The constant added to force real significance of main effect A was 0.25.

Adjusted Original Effect Significant

.05 Level of Significance

Significance tests of main effect A showed that more than 99% of all significant records remained significant after all transformations except 07 (Inverse). Only 73.32% remained significant after transformation 07. This pattern continued for all main effects and interactions; 24-32% remained significant at the .05 level of significance. For effects other than A, transformations 05 (LOG_{10}) and 08 (LOG_{ε}) caused the percentage of significant records to drop to between 59 and 64% while all other transformations left at least 78% significant.

.01 Level of Significance

Significance tests of main effect A showed that more than 95% of all significant records remained significant after all transformations except 07 (Inverse). Only 40.57% remained significant after transformation 07. This pattern continued for all other main effects and interactions; 8-20% remained significant at the .01 level of significance. For all effects other than A, transformations 05 (LOG_{10}) and 08 (LOG_{ε}) caused the percentage of significant records to drop to between 42 and 58% while all other transformations left at least 70% significant.

Adjusted Original Effect Not Significant

.05 Level of Significance

Effects other than A were changed to significant less than 4% of the time by all transformations. Main effect A was affected more than 4% of the time by all transformations except 09 (SINE).

.01 Level of Significance

Effects other than A were changed to significant less than 2% of the time by all transformations. Transformation 06 (EXP(x)) changed 12.36% to significant on main effect A. All other transformations on A affected less than 5% of the records.

BINOMIAL DISTRIBUTION RESULTS

A random variable X is said to be distributed according to the Binomial Distribution if the probability function is given by

$$f(x) = \binom{n}{x} p^{x} q^{n-x}$$

where $p + q = 1$ and $\binom{n}{x} = \frac{n!}{x!(n-x)!}$. The parameters used were Trials = $n = 6$ and Probability of Success = $p = 0.5$. The constant added to force real significance of main effect A was 1.50. To enable all transformations to be performed, 8578 observations (1.05% of the total number) that were found to be zero after the addition of the constant were changed to 1.0.

Adjusted Original Effect Significant

.05 Level of Significance

Significant tests of main effect A showed that more than 97% of all significant records remained significant after all transformations except 09 (SINE) and 10 (HYPERBOLIC TANGENT). Only 85.27% remained significant after transformation 09; 56.74% after transformation 10. This pattern continued through all other main effects and interactions with transformations 09 and 10 causing less than 41% and 30% respectively to remain significant. For all effects other than A, only transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x} + \sqrt{x+1}$) left more than 80% significant.

.01 Level of Significance

Significance tests on main effect A showed that four of the transformations caused less than 93% of the significant records to remain significant. They were 06 (EXP(x)) with 84.73%, 07 (INVERSE) with 87.85%, 09 (SINE) with 65.48%, and 10 (HYPERBOLIC TANGENT) with only 15.78% remaining significant. For effects other than A, only transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x} + \sqrt{x+1}$) left more than 75% significant.

Adjusted Original Effect Not Significant

.05 Level of Significance

Effects other than A were changed to significant less than 6% of the time by all transformations. Main effect A was changed to significant more than 6% of the time by all transformations except 03 ($\sqrt{x}$) and 04 ($\sqrt{x} + \sqrt{x+1}$).

.01 Level of Significance

Effects other than A were changed to significant less than 3% of the time by all transformations. Main effect A was changed to significant less than 6% of the time by all transformations except 06 (EXP(x)) which changed 12.37%.

ERLANG DISTRIBUTION RESULTS

The Erlangian distribution of order k has as its probability density function:

$$f(t) = \frac{\lambda}{\Gamma(k)} (\lambda t)^{k-1} \varepsilon^{-\lambda t}.$$ (Parzen, 1962, p. 199).

The parameters used were mean = 4 and k = 3. There were 139 observations (.017% of the total number) greater than 18.0 which were changed to 18.0. The constant added to force real significance of main effect A was 2.0.

Adjusted Original Effect Significant

.05 Level of Significance

Significance tests of main effect A showed that more than 91% of all significant records remained significant after six of the transformations. However, transformation 06 (EXP(x)) left only 6.58% significant. Transformation 09 (SINE) left 30.65% significant and 10 (HYPERBOLIC TANGENT) left 41.18% significant. For effects other than A, only transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x}$ + $\sqrt{x+1}$) left more than 78% of the records significant.

.01 Level of Significance

Significance tests of main effect A showed that more than 95% of all significant records remained significant after five of the transformations. Transformation 06 (EXP(x)) left only 1.76% significant; 07 (INVERSE) left 69.77% significant; 09 (SINE) left 12.29% significant; and 10 (HYPERBOLIC TANGENT) left 10.18% significant. For effects other than A, only transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x+1}$ left more than 75% significant (except for effect ABC where only 67% were left significant). Throughout all effects, transformations 06, 09, and 10 caused the greatest loss of significance.

Adjusted Original Effect Not Significant

.05 Level of Significance

Effects other than A were changed to significant less than 6% of the time by all transformations. Main effect A was changed more than 25% of the time by all transformations except 06 (EXP(x)) which only changed .41%.

.01 Level of Significance

Effects other than A were changed to significant less than 2% of the time by all transformations. Main effect A was changed more than 16% of the time by all transformations except 06 (EXP(x)) which changed none, and 10 (HYPERBOLIC TANGENT) which changed 5.09%.

EXPONENTIAL DISTRIBUTION RESULTS

A continuous random variable X assuming all non-negative values is said to have an Exponential distribution with parameter $\alpha > 0$ if its probability density function is given by

$$f(x) = \alpha\varepsilon^{-\alpha x}, \quad x>0$$
$$= 0 \quad \text{elsewhere.}$$

Meyer, 1965, p. 173. The parameter used was mean = 2. There were 108 observations (.013% of the total number) greater than 18.0 which were changed to 18.0. The constant added to force real significance of main effect A was 2.0.

Adjusted Original Effect Significant

.05 Level of Significance

Significance tests of main effect A showed that only three transformations changed any significant values. Transformation 06 (EXP(x)) left only 5.21% significant; 07 (INVERSE) left 35.67% significant; and 09 (SINE) left 66.25% significant. For effects other than A, only transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x}$ + $\sqrt{x+1}$) left at least 70% significant. Transformations 06 and 07 left less than 8% significant for all effects other than A.

.01 Level of Significance

Significance tests of main effect A showed that only three transformations left less than 99% significant. They were transformation 06 (EXP(x)) which left only 1.39% significant; 07 (INVERSE) which left 13.54% significant; and 09 (SINE) which left 43.96% significant. For effects other than A, only transformation 04 ($\sqrt{x}$ + $\sqrt{x+1}$) left at least 70% significant (except on effect ABC which was only 57%). As in the .05 level of significance above, transformations 06 and 07 left the least percentage of significant records.

Adjusted Original Effect Not Significant

.05 Level of Significance

Effects other than A were changed to significant less than 7% of the time by all transformations. Main effect A was changed more than 31% of the time by all transformations except 06 (EXP(x)) which changed none.

.01 Level of Significance

Effects other than A were changed to significant less than 2% of the time by all transformations. Main effect A was changed more than 18% of the time by all transformations except 06 (EXP(x)) which changed none and 07 (INVERSE) which changed 9.21%.

GAMMA DISTRIBUTION RESULTS

A random variable X is said to be distributed according to the Gamma distribution if its density function is given by

$$f(x) = \frac{1}{\Gamma(\alpha+1)\beta^{\alpha+1}} \quad X^{\alpha}\varepsilon^{-X/\beta} \quad 0<X<\infty$$

where α and β are parameters with $\alpha > -1$ and $\beta>0$. (Beyer, 1966, p. 18). The parameters used were mean = 4 and k = 3. There were 130 observations (.016% of the total number) greater than 18.0 which were changed to 18.0. The constant added to force real significance of main effect A was 2.0.

Adjusted Original Effect Significant

.05 Level of Significance

Significance tests of main effect A showed that more than 91% of all significant records remained significant after six of the transformations. However, transformation 06 (EXP(x)) left only 6.26% significant. Transformation 09 (SINE) left 30.55% significant and 10 (HYPERBOLIC TANGENT) left 40.26% significant. For effects other than A, only transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x}$

+ $\sqrt{x+1}$)left more than 75% of the records significant.

.01 Level of Significance

Significance tests of main effect A showed that more than 95% of all significant records remained significant after five of the transformations. Transformation 06 (EXP(x)) left only 1.58% significant; 07 (INVERSE) left 69.29% significant; 09 (SINE) left 11.71% significant; and 10 (HYPERBOLIC TANGENT) left 8.90% significant. For effects other than A, only transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x}$ + $\sqrt{x+1}$) left at least 70% significant. Throughout all effects, transformations 06, 09, and 10 caused the greatest loss of significance.

Adjusted Original Effect Not Significant

.05 Level of Significance

Effects other than A were changed to significant less than 6% of the time by all transformations. Main effect A was changed more than 21% of the time by all transformations except 06 (EXP(x)) which only changed .10%.

.01 Level of Significance

Effects other than A were changed to significant less than 2% of the time by all transformations. Main effect A was changed more than 15% of the time by all transformations except 06 (EXP(x)) which changed none and 10 (HYPERBOLIC TANGENT) which changed 5.06%.

LOG NORMAL RESULTS

A random variable X is said to be distributed according to the Log-normal distribution if its probability density function is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} X^{-1}\varepsilon^{-(\ln x-\alpha)^2/2\beta^2} \quad X>0,\ \beta>0,\ \alpha\geq 0$$

$$= 0 \qquad \text{elsehwere}$$

(Miller, 1965, p. 77). The parameters used were mean = 4 and standard deviation = 1. The constant added to force real significance of main effect A was 2.00.

Adjusted Original Effect Significant

.05 Level of Significance

Significance tests of main effect A showed that more than 90% of all significant records remained significant after all transformations except 09 (SINE). Transformation 09 left 20.59% significant. For all other effects, transformation 09 caused the greatest drop in significance. For effects other than A, only transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x}$ + $\sqrt{x+1}$) left at least 85% significant.

.01 Level of Significance

Significance tests of main effect A showed that six of the transformations had no effect. Transformation 06 (EXP(x)) left 78.74% significant; 09 (SINE) left only 7.76% significant; and 10 (HYPERBOLIC TANGENT) left 70.12% significant. For effects other than A, only transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x}$ + $\sqrt{x+1}$) left at least 80% significant. Transformation 09 caused the greatest drop in significance.

Adjusted Original Effect Not Significant

.05 Level of Significance

Effects other than A were changed to significant less than 6% of the time by all transformations except 10 (HYPERBOLIC TANGENT). Transformation 10 changed approximately 28% of all records on effects other than ABC.

.01 Level of Significance

Effects other than A were changed to significant less than 2% of the time by all transformations except 10 (HYPERBOLIC TANGENT). Transformation 10 changed approximately 11% of all records on effects other than ABC.

NORMAL DISTRIBUTION RESULTS

A random variable X is said to be distributed according to the Normal distribution if its density function is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\ \varepsilon^{-(x-\mu)^2/2\sigma^2} \quad -\infty < x < \infty.$$

(Beyer, 1966, p. 18). The parameters used were mean = 6 and standard deviation = 1. The constant added to force real significance of main effect A was 3.0.

Adjusted Original Effect Significant

.05 Level of Significance

Significance tests on main effect A showed that more than 97% of all significant records remained significant after all transformations 09 (SINE). Only 67.53% remained significant after transformation 09. For effects other than A, transformations 03 ($\sqrt{x}$), 04 ($\sqrt{x}$ + $\sqrt{x+1}$), 05 (LOG_{10}), and 08 (LOG_{ε}) left at least 76% of the records significant. Transformation 09 continued to cause the greatest drop in significance for all effects (except ABC which was reduced to zero significant records by transformation 10 (HYPERBOLIC TANGENT)).

.01 Level of Significance

Significance tests on main effect A showed that more than 96% of all significant records remained significant after all transformations except 09 (SINE). Only 42.36% remained significant after transformation 09. For effects other than A transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x}$ + $\sqrt{x+1}$) left at least 76% of the records significant. Effect ABC had all significant records changed by transformation 10 (HYPERBOLIC TANGENT) which left 67-97%.

Adjusted Original Effect Not Significant

.05 Level of Significance

Effects other than A were changed to significant less than 8% of the time by all transformations except 10 (HYPERBOLIC TANGENT). Transfortion 10 changed no records on effect ABC but changed 69-99% of records for other effects.

.01 Level of Significance

Effects other than A were changed to significant less than 4% of the time by all transformations except 10 (HYPERBOLIC TANGENT). Transformation 10 changed no records on effect ABC but changed 67-98% of records for other effects.

POISSON DISTRIBUTION RESULTS

A random variable X is said to be distributed according to the Poisson distribution if its probability function is given by

$$f(x) = \frac{\varepsilon^{-m} m^{x}}{x!} \quad m > 0, 1, 2, \cdots$$

(Beyer, 1966, p. 19). The parameter used was mean = 4. The constant added to force real significance of main effect A was 2.0. To enable all transformations to be performed, 9972 observations (1.23% of the total number) that were found to be zero after the addition of the constant were changed to 1.0.

Adjusted Original Effect Significant

.05 Level of Significance

Significance tests of main effect A showed that more than 97% of all significant records remained significant after six of the transformations. However, transformation 06 (EXP(x)) left only 26.31% significant; 09 (SINE) left 8.55% significant; and 10 (HYPERBOLIC TANGENT) left 36.39% significant. For effects other than A, the above three transformations caused the greatest loss of significance. Transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x} + \sqrt{x+1}$) were the only ones to leave more than 80% significant on all effects.

.01 Level of Significance

Significance tests of main effect A showed that at least 90% of all significant records remained significant after five of the transformations. Transformation 06 (EXP(x)) left only 9.78% significant; 07 (INVERSE) left only 79.91% significant; 09 (SINE) left 2.04% significant; and 10 (HYPERBOLIC TANGENT) left 9.30% significant. The above four transformations caused the greatest loss of significance for all effects. Transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x} + \sqrt{x+1}$) were the only transformations to leave at least 68% of all effects significant.

Adjusted Original Effect Not Significant

.05 Level of Significance

For all effects other than A, less than 7% of the records were changed by all transformations. For main effect A, all transformations changed more than 15% of the records except 06 (EXP(x)).

.01 Level of Significance

For all effects other than A, less than 3% of the records were changed by all transformations. For main effect A, all transformations changed more than 12% of the records except 06 (EXP(x)) which changed none; 09 (SINE) changed 3.55% of the records and 10 (HYPERBOLIC TANGENT) changed 3.95% of the records.

UNIFORM DISTRIBUTION RESULTS

A random variable is said to be distributed uniformly over the interval [a,b], where both a and b are finite, if its probability density function is given by

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$
$$= 0, \quad \text{elsewhere}$$

(Meyer, 1965, p. 64). The parameters used were lower bound = a = 1 and upper bound = b = 5. The constant added to force real significance of main effect A was 1.0.

Adjusted Original Effect Significant

.05 Level of Significance

Significance tests of main effect A showed that more than 97% of all significant records remained significant after all transformations except 09 (SINE) and 10 (HYPERBOLIC TANGENT). Transformation 09 left 61.26% significant; 10 left 80.68% significant. For effects other than A, the above two transformations caused the greatest drop in significance. For all effects only transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x} + \sqrt{x+1}$) consistently left at least 85% of the records significant.

.01 Level of Significance

Significance tests of main effect A showed that five of the transformations left more than 95% of the records. However, transformation 02 (ARC TANGENT) left 88.39% significant; 07 (INVERSE) left 84.24%; 09 (SINE) left 40.70%; and 10 (HYPERBOLIC TANGENT) left 36.52%. For effect other than A, only transformations 03 ($\sqrt{x}$) and 04 ($\sqrt{x} + \sqrt{x+1}$) left more than 79% of the records significant.

Adjusted Original Effect Not Significant

.05 Level of Significance

For effects other than A, less than 6% of the records were changed by all transformations. Main effect A was changed more than 6% of the time by all transformations except 09 (SINE) which only changed .10%.

.01 Level of Significance

For effects other than A, less than 3% of the records were changed by all transformations. Main effect A was changed more than 6% of the time by all transformations 02 (ARC TANGENT), 06 (EXP(x)), 07 (INVERSE), and 10 (HYPERBOLIC TANGENT).

HIGHWAY DISTRIBUTION RESULTS

The distribution was generated from actual highway accident damage costs as estimated by investigating officers. The observations were obtained by reading the Alabama Highway Department accident file. Each observation was divided by 1000 so amounts could be transformed by 06 (EXP(x)). A frequency table of observation (expressed in thousands of dollars) is found in Table 1. After the above coding, 353 observations (.05% of the total number) were found to be greater than 18.0 and were changed to 18.0. There was no constant added in this distribution to force significance.

Adjusted Original Effect Significant

.05 Level of Significance

Significance tests on all main effects and interactions showed that only transformations 02 (ARC TANGENT), 03 ($\sqrt{x}$), 04 ($\sqrt{x+1}$) and 10 (HYPERBOLIC TANGENT) left at least 78% of the records significant. The other five transformations caused a much greater drop in significance. Transformation 07 (INVERSE) caused the greatest drop in significance.

.01 Level of Significance

Significance tests of all main effects and interactions showed that only transformations 02 (ARC TANGENT), 03 ($\sqrt{x}$), 04 ($\sqrt{x} + \sqrt{x+1}$) and 10 (HYPERBOLIC TANGENT) left at least 74% of the records significant. Transformation 07 (INVERSE) left less than 6% of the records significant.

Adjusted Original Effect Not Significant

.05 Level of Significance

All effects were changed less than 11% of the time by all transformations. Transformation 05 (LOG_{10}) and 08 (LOG_{ϵ}) caused the greatest changes.

.01 Level of Significance

All effects were changed less than 5% of the time by all transformations.

SUMMARY

Nine transformations were studied for effect on the analysis of variance. They were ARC TANGENT, $\sqrt{x}$, $\sqrt{x} + \sqrt{x+1}$, LOG_{10}, EXP(x), INVERSE, LOG_{ϵ}, SINE AND HYPERBOLIC TANGENT.

The factorial experiment used as a basis for this study was a three factor factorial with three levels per factor and three observations per cell. One effect was altered by the addition of a constant to provide one effect with a high percentage of significant records.

The transformations were applied to observations distributed according to the following distributions: Beta, Binomial, Erlang, Exponential, Gamma, Log-normal, Normal, Poisson, Uniform and actual data from highway damage costs. For all distributions except the highway accident damage costs (8436), 10,000 experiments were studied.

When analysis of variance results were compared (before and after transformation), $\sqrt{x}$ and $\sqrt{x} + \sqrt{x+1}$ consistently changed the fewest results. All other transformations produced unstable results across the range of distributions, showing little effect to sometimes drastic change according to the distribution involved.

BIBLIOGRAPHY

1. Beyer, William H., Ed. CRC Handbook of Tables for Probability and Statistics. Cleveland, Ohio: Chemical Rubber Co., 1966.

2. Davies, Owen L., Ed. Design and Analysis of Industrial Experiments. London: Oliver and Boyd, 1954.

3. Dixon, W. J., Ed. Biomedical Computer Programs. Los Angeles: University of California Press, 1973.

4. Freyer, H. C. Concepts and Methods of Experimental Statistics. Boston: Allyn & Bacon, 1966.

5. International Business Machines, IBM OS Full American National Standard Cobol. White Plains, New York: IBM, 1972.

6. Kiviat, P. J., Villaneuva, R.; and Markowity, H. M. SIMSCRIPT II.5 Programming Language. Edited by E. C. Russell. Arlington, Virginia: Consolidated Analysis Centers, 1973.

7. Meyer, Paul L. Introductory Probability and Statistical Applications. Reading, Mass.: Addison-Wesley Publishing Co., 1965.

8. Miller, Irwin, and Freund, John E. Probability and Statistics for Engineers. Englewood Cliffs, New Jersey: Prentice Hall, 1965.

9. Norton, Dee Wiswall. "An Empirical Investigation of the Effects of Normality and Heterogenity Upon the F-Test of Analysis of Variance." Ph.D. Dissertation, State University of Iowa, 1952.

10. Parzen, Emanuel. Stochastic Processes. San Francisco: Holden-Day, 1962.

11. Struble, George. Assembler Language Programming: The IBM System/360. Reading, Massachusetts: Addison-Wesley Publishing Co., 1969.